Extending and redefining Attitude and other concepts in the context of Mathematics Education

Authors: Anita Stark and Agnete K. Ravnborg

Address:

Slæggerupvej 107
4000 Roskilde
Denmark
E-mail address:
astark@alfa-og-omega.dk

Pilekrattet
2630 Taastrup
Denmark
E-mail address:
toga@ruc.dk

Conceptual Foundations

In what follows, definitions of the concepts belief, emotion mood and attitude are developed.
In developing these definitions, psychological and social concepts, among other things, have been used. The psychological terms which most often turn up in research on mathematical didactics are extended by defining them broader and by incorporating the individual's interaction with his surroundings. Inspiration for this extension has in particular come from Bourdieu's concept of Habitus.

The Affective Domain

We have desired to base ourselves on ideas and research developed in the affective domain of mathematical didactics. According to the survey paper by McLeod: 'Research on Affect in Mathematics Education: A Reconceptualisation' (McLeod, 1992), the affective domain consists of beliefs about, feelings for and attitudes towards mathematics, learning and education. He writes that the affective domain:

> *…refers to a wide range of beliefs, feelings and moods that are generally regarded as going beyond the domain of cognition… beliefs, attitudes and emotions … will be used … as more specific descriptors of subsets of the affective domain (McLeod, 1992:576).*

We have chosen to take as our starting point McLeod's subdivision of the affective domain. In this, a belief, an emotion or an attitude are all denoted an *affective aspect*. McLeod's definitions of the various affective aspects are illustrated in Table 1 below.

	Beliefs	**Attitudes**	**Emotions**
Duration of an affective aspect	Long	Intermediate between long and short	Short
Intensity of an affective aspect	Low	Intermediate	High
Time consumed in developing an affective aspect	Long duration	Intermediate between long and short duration	Short duration
The cognition content in an affective aspect	Large cognition content	Some cognition content	Very little cognition content

TABLE 1: This is our listing of McLeod's categories of beliefs, attitudes and emotions (McLeod, 1992:578)

The McLeod definitions of beliefs, emotions and attitudes are based on a theory put forward by Mandler, a researcher in cognitive psychology. Mandler's theory deals with what happens after the disruption of a conceived plan. For example, the solution of a time-demanding task may be seen as a disruption of the plan 'all tasks may be completed inside a few minutes'. Compare (Svege, 1995:9). When the plan is broken off, an intense emotion may be felt. Repeated intense emotional reactions of short duration may over time develop into more stable and less intense attitudes. Starting out with this piece of theory, McLeod defines and ranges the three categories: beliefs, attitudes and emotions that he then applies in order to give structure to the vast field that the affective domain constitutes within mathematics, learning and education. Cf. (Mandler, 1989, interpreted in McLeod 1992:577 – 578, Svege, 1995:8 – 11).

The description of the affective aspects contains a contradiction, since they on the one hand are seen as 'going beyond the domain of cognition', whereas on the other hand the content of cognition aids in differentiating the aspects themselves. Using McLeod's definitions (see again Table 1), the categorisation of the affective domain is characterised by a dichotomous build-up, containing oppositions between cognition – affection, eternal – ephemeral, and great intensity – low intensity. Table 1 demonstrates that McLeod puts emotions in opposition to beliefs and that he defines the two concepts via a string of dichotomies. According to Eysenck and Fog, feelings may be perceived as temporary when they are seen as divorced from reason, i.e. when emotion is seen as being in opposition to rationality. Cf. (Eysenck, 1990:129, Fog, 1992:42). McLeod's terms are defined in terms of the dichotomy between reason and emotion, corresponding to the dichotomy between cognition and affection.

Dichotomy

Bishop describes various cultural values attached to reference-free mathematics, the form of mathematics usually taught in the Western world. The values rationality and objectiveness are seen as core values of this form. Bishop regards the two values as a complementary couple. The value *rationality* implies that whatever is not rational must be regarded as untrustworthy (Bishop, 1991:201 – 202). When rationality is added to the dichotomy sense / emotion, sense is perceived as things rational thought out in the mind and emotion as the irrational experienced by the body. Cf. (Damasio, 1994: 248, Fog, 1992: 42 – 45).

According to P. J. Davis mathematical statements are usually perceived thus:

> *… that such a [mathematical] statement is perfect in its precision and in its truth, is absolute in its objectivity, is universally interpretable, is eternally valid, and expresses something that must be true in this world and in all possible worlds What is mathematical is certain (Davies, 1993:183).*

It is debatable whether mathematics is rational, objective and eternal, but the teaching of mathematics and research in the way mathematics is taught are not, because human relations and studies of relations are investigated.

In the world of didactics one recognises the phenomenon of ascribed values that occur in the world of mathematics. The rational and eternal as opposed to the emotional and the transitory.

In McLeod's article, a substantial part of the work is done on the main category Beliefs; within two other related categories, 'related concepts from the affective domain' and ' topics related to the cognitive domain', a large part of the research refers back to 'Beliefs about Self' (McLeod, 1992:579 – 582). Thus an important part of research in the affective domain is naturally related to the most cognitive aspect,

beliefs. The above leads to the interpretation, that researchers regard beliefs as more important than emotions. In general, beliefs are valued higher than emotions.

Aesthetics is discussed as an emotion and categorised as a related concept which belongs to the cognitive domain. McLeod has focused on cognition, having chosen to describe the emotion aesthetics in a context which stresses cognition. Cf. (McLeod, 1992:578, 586). He links aesthetics with cognition (McLeod, 1992:578). One argument for doing this may be that aesthetics imply emotions that accompany and influence the thinking of a mathematician who works with a problem in his field. McLeod places phobia related to mathematics in line with related concepts of the affective domain (See McLeod, 1992:584). It is an intriguing feature of this paper that seemingly similar aspects are categorised in different ways. Positive aspects are often viewed as a belief and negative aspects as emotions.

McLeod points out, that research in the affective domain is in need of a stronger theoretical foundation (McLeod, 1992:590). In modern cognitive research, affective aspects are often dispensed with (McLeod, 1992:577), because they are very difficult to describe and measure (McLeod, 1992:576). It's hard to endow them with an objective description. His bid for how such a theory may be developed is as follows: 'studies of affect must be integrated with studies of cognition' (McLeod, 1992:588). Cf. (McLeod, 1992:575, 589).

We see McLeod's quest for a unifying theory as the expression of a desire to place the affective aspects in a cognitive framework. One such example of a cognitive structure is the view of the foundations of mathematics that is described by Triadafillidis (It is a point of view on which Triadafillidis himself is not in agreement, but which he nevertheless sees presented in the world of mathematics)..:

> *Just as the existence of God does not require the existence of the world, so the existence of mathematics does not depend on its having earthly origins. Thus began a mathematisation of the discipline of mathematics itself, which would eventually lead to divorcing mathematics from the sphere of senses, feelings, intuition, and non-exact practises (Triadafillidis, 1998:21).*

Here mathematical science is described as removed from the affective domain. Mathematics, described by images that contain the divine as well as remote principles that underlie everything, thereby turns into an expression of ascribed values in the cognitive domain.

McLeod's definitions of the concepts beliefs, emotions and attitudes spring from a basically dichotomous conception of man, where the thinking head is severed from the feeling body. As a consequence, human beings are perceived in a fundamental way as split in two (Damasio, 1994:248).

Affective aspects should be credited with value

It is our belief that the definitions of the three affective aspects should encompass everything that is placed under the heading affective domain of mathematics. McLeod's definitions are too limited to comprehensively describe the entire field, since some important aspects that relate to human beings in connection with mathematics cannot be made to tally with these definitions. As an example, one may mention emotions and beliefs of longer duration; these are only slightly cognitive. For this reason we have chosen to expand and change McLeod's definition of the categories beliefs, emotions and attitudes.

Svege has applied the McLeod definitions of the affective domain in her paper: 'Affektive sider ved undervisning og studenters læring av matematikk' (Svege, 1995). Svege extends the concept beliefs to incorporate unconscious beliefs. In her analysis longer-lasting emotions are included; however the

question whether they are emotions or not in accordance with the definition of this term is never put. As a consequence, the deficiencies relating to the definitions themselves don't become apparent in Sveges work. Cf. (Svege, 1995:3, 54 – 85).

Jeff Evans defines the concepts in the same way as McLeod does (page 44), yet they are not applied in accordance with McLeod's original definitions in his work. The author of this article finds that the concepts don't seem to be applied in line with given definitions. An example:
All interviewees describe experiencing feelings in connection with their Mathematics activities. Evans has been on the lookout for anxiety and fear as well as confidence, pleasure and anger. In this context Evans defines all of these as emotions (page 171). However, on page 12 of the study confidence is described as an attitude, the same goes for familiarity (at – homeness). The basic definitions therefore seem to be inconsistent. We feel that the reason for this type of mix-up lies in inadequate and dysfunctional definitions of basic concepts. This being said, it must be stressed that when Evans focuses his attention on anxiety, confidence and similar phenomena, it is entirely appropriate and serves to demonstrate that emotions form a large part of human attitudes towards Mathematics.

We wish to add to the whole field a theoretical framework that confers upon affection a value equal to that of cognition. The reason for this is that we believe the present framework has negative implications for the teaching of mathematics.

It will take a fundamentally different outlook if the dichotomy between sense and sensibility is to be overcome. It won't suffice merely to ask more questions within the dichotomy, because questions founded upon opposites are bound to yield answers that uphold and cement this very opposition. Cf. (Fog, 1992:66).

It is our fundamental belief that feelings and rationality are indivisible and mutually connected. This belief points towards assigning to both a positive value, thereby avoiding a dichotomous division of things into that, which is valuable as opposed to what has no value. This fundamental belief is a good starting point for the elaboration of a theory that a priori confers on all constituents of the affective domain a positive value.

Definition of concepts

It is man as a *whole being* who has a relationship with mathematics. By the term 'whole', one understands the notion that man's head cannot be separated from his body.
When human beings are seen as whole entities that cannot be split into two independent parts, man in his entirety is assigned value. This basic view of individuals and their thinking is in correspondence with Damasios' views. He writes that a complete understanding of human thought demands a point of view which includes the body. Thoughts must not only be transferred from a non-physical existence to an existence in the real world that consists in biological tissue. They must also be brought to relate to the body in its entirety, that is participate in the integration of body and mind, and they must interact with the physical and social environment (Damasio, 1994:254). In order to examine the affective aspects of 'human interrelation with mathematics' given the point of departure that pupils are whole human beings, we have developed our own definitions of beliefs, emotions and attitudes.

Beliefs

According to Pehkonen, knowledge is objective and free of influences from affective aspects (Pehkonen, 1998:50). A belief is defined as knowledge that is not necessarily well-founded nor

logically true when seen in relation to an individual's other beliefs and knowledge (Pehkonen, 1998:49). McLeod and Pehkonen both regard beliefs as preponderantly cognitive. Here we perceive a contradictary relationship between Pehkonen's two ways of defining knowledge, it being spoken of as something objective as well as something not necessarily well-founded. Furthermore he writes that it is difficult to conceptually clarify beliefs, since a clear-cut distinction between beliefs and knowledge is impossible (Pehkonen, 1998:41). Some researchers put forward the argument that the interesting aspect really is to study the combined influences from beliefs and knowledge on the learning situation (Pehkonen, 1998:49).

Knowledge is acquired and exists in conjunction with affective aspects, since the acquisition of knowledge does not happen in a vacuum. Therefore one does not have to make a distinction between beliefs and knowledge in order to investigate human beings' interaction with mathematics.

Since we desire to investigate whole human beings' interaction with mathematics, the definition of beliefs must necessarily include beliefs that are very cognitive as well as those that are only slightly cognitive. Otherwise it will not be possible to investigate all types of beliefs; an example could be a belief that has developed without it being possible to explicitly put it in words.

C. Hasse makes use of the concepts implicit and explicit beliefs:

1) Explicit beliefs, those that are highlighted as forming relevant causal relationships and vocalised explanations in conversation.

2) Implicit beliefs: these are often not put forward as causal links in spoken conversation, but enter into it as a silent (non-verbalised) knowledge…(Hasse, 1998:7).

Whenever pupils who don't do well in school express the idea that failure at mathematics is due to their own insufficiency, they implicitly voice the belief that 'I'm the one responsible for my learning mathematics'. Cf. (MacLeod, 1987:113, as referred to by Frankenstein, 1995:167)).

We define *beliefs* as conceptions of (or knowledge about) the world. These beliefs evolve in a continued interrelation between an individual and the surroundings, and they are imbedded in him or her. Beliefs may be explicit or implicit. Both types of beliefs are expressible via speech and behaviour.
'It brings more status when you are good at mathematics than when you excel in other subjects' is an example of an explicit belief (Svege, 1995:83). Whenever a political decision is discussed on the basis of a mathematical model, the debate often turns on 'correct' or 'false', rather than 'ethical' or 'unethical' (Christiansen, 2000:62). In this case we deem that somewhere in the background there is lurking an implicit belief held by the participants in the discussion that mathematical arguments have greater value than other arguments.

Emotions

There are various definitions of emotions (feelings) in use. A characteristic of this concept is that it is not well-defined. Goleman speaks of emotions as

> *…a term, the precise meaning of which has caused the splitting of hairs between psychologists and philosophers for more than a century (Goleman, 1997:389).*

According to Goleman, Paul Ekman claims that four core emotions: fear, anger, sadness and joy are to be universally found all over thee world (Ekman, 1992, as referred to in Goleman, 1997:390).

In psychology, emotions are often defined as being of short duration only. In the encyclopaedia on Psychology, 'Fra angst til ånd (From Fear to Spirit) ' (Abrahamowitz, 1999), emotions are described as short-term changes in a basic mood. Something is happening to us, and this is perceived by our senses. At the same time alterations occur in the organism, and we react to these changes. For example, according to this definition, spontaneous joy is viewed as an emotion, whereas moods that last don't qualify as such. Confer here also (Ekman, 1992 referred to in Goleman, 1997:394)). This corresponds to McLeod's idea of emotions. In our opinion, emotions are defined as short-duration phenomena exclusively on the backdrop of the dichotomous perception of man. When emotions are pinned down as short-lived and at times ephemeral, it is because they are regarded as the opposite of reason, in turn seen as objective and hence lasting. Cf. (Eysenck, 1990:129, Fog, 1992:42).

When the basic view of man consists in the idea that feelings and thought are connected, the value of emotions for a whole person are recognised. Since we wish to investigate the interrelation of whole human beings with mathematics, the definition of emotions must necessarily include both longer-lasting and shorter-lasting emotions. Otherwise it will not be possible to investigate those emotions of long duration that form a part of, say, fascination for and fear of mathematics.

We define *emotions* as experiences that are embedded in a human being via the senses. An emotion implies a distinct physical reaction of the body. Emotions may be of short duration or long duration. An example of a short-lived emotion is a so-called 'Aha – experience' in connection with the solution of an exercise (McLeod, 1992:582). An example of an emotion which may be described as lasting, is the deep-felt elation over mathematics expressed by the teacher Carsten Cramon, MSc, in the following quote:

> *Mathematics has been my great passion all through my life. This science represents to me an amazingly beautiful universe. Cramon as quoted in (Amnitzbol, 2000:19).*

Attitudes

The definition of attitudes is presented in four different ways in Psychology, with four different points of departure. The starting point may be either beliefs, emotions, propensities or all three concepts in conjunction (Eysenck, 1990:30). The McLeod definition of an attitude starts out with emotions, and attitudes are defined as something in between a belief and an emotion. The disposition to prefer something over something else is given as an example of an attitude (McLeod, 1992:578, Eysenck, 1990:30). Table 1 may also be referred to in this context.

Since, as already stated, we wish to make whole human beings our starting point, McLeod's definition of attitudes conflicts with our fundamental point of view. When one considers a definition of attitude that is based on combining all three concepts, beliefs and emotions are no longer necessarily regarded as opposites, hence such a definition is better suited to views.

Eysenck, Abrahamowitz and Pehkonen all define an attitude as the combination of the three concepts: belief, emotion and propensity, the latter not always converted into actual action (Eysenck, 1990:30, Abrahamowitz, 1999, Pehkonen, 1998:44).
A propensity is the driving force behind a person's actions as, say, an intention or an aspiration (Beyer, 1997). These authors regard the three elements as phenomena that occur in individuals, and attitude is viewed as that which starts off the actions of an individual.

In order to illuminate human beings' interaction with mathematics, a concept which contains their actions as well as their intended actions is lacking. In accordance with our basic view of man we don't regard propensity as isolated from surroundings, that is, as existing solely inside the human being in question.
The concept *propensity* is extended to include an individual's scope of (possible) actions, as well as the (actuated) action and the inclination to act (passive action) itself.

Propensity is expressed in an individual's interaction with his surroundings.

By definition, we take the concept *attitude* to mean a belief and an emotion that together cause a propensity.
The *propensity* delineates a restricted set of possible actions. The actuated or passive action thereby becomes the concrete expression of an attitude.

The individual and his surroundings

We have worked out an illustration that depicts a model of our view of the interaction between an individual and his surroundings.

The concept attitude has been defined above as a belief and an emotion that together give rise to a propensity. In this way the affective domain contains all aspects that form part of attitudes.

Attitudes make up a part of human beings' interactions with surroundings, and they are decisive for the nature of this interaction, because the actuated actions of an individual (chosen from the propensity) make up the interrelation between this individual and his surroundings.
The attitudes of individuals are embedded in the body and they show themselves in the various relationships she/ he finds herself/ himself involved in. Attitudes are continuously developed in this interaction with surroundings. Our perception of attitudes is founded on an idea of human beings as whole individuals who think, feel and act at the same time.

The four situations are found in fig.1, where there are various interrelations with

- Family
- School
- Leisure time
- Society as a whole

The different relations contain, say, possibilities for and limitations to the free expression of an individual. The possibilities and limitations are to be found in the attitudes of the individual, among other things.

Biographical note

Anita Stark is self-employed and owns a business: alfa og omega address: Slæggerupvej 107, 4000 Roskilde, Denmark <astark @alfa-og-omega.dk>. Main interests: social issues and working for a better quality of life in relation to math education as well as in other areas. I spend my time applying research results to the real world by working hands-on with children and teenagers as well as grown-ups who experience various problems when it comes to learning mathematics.

Agnete K. Ravnborg is currently on sick leave following an accident. Address: Pilekrattet, 2630 Taastrup, Denmark toga@ruc.dk. Main interests: improving the quality of life for people in precarious situations.

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